UNIVERSIDADE FEDERAL DO RIO DE JANEIRO

INSTITUTO COPPEAD DE ADMINISTRAÇÃO

MATEUS ALVES MARTINS PORTELINHA

THE IMPACTS OF CRYPTOCURRENCIES IN THE PERFORMANCE OF BRAZILIAN STOCKS' PORTFOLIOS

Rio de Janeiro

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Master's dissertation presented to the COPPEAD Graduate School of Business, Universidade Federal do Rio de Janeiro, as part of the mandatory requirements in order to obtain the title of Master in Business Administration (M.Sc.).

Supervisor: Prof. Carlos Heitor Campani, Ph.D.

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CIP - Catalogação na Publicação

PP843t

Portelinha, Mateus Alves Martins
The impacts of cryptocurrencies in the performance of Brazilian stocks' portfolios / Mateus Alves Martins Portelinha. -- Rio de Janeiro, 2021.
38 f.

Orientador: Carlos Heitor Campani.
Dissertação (mestrado) - Universidade Federal do Rio de Janeiro, Instituto COPPEAD de Administração, Programa de Pós-Graduação em Administração, 2021.

1. Cryptocurrencies. 2. Optimal portfolio strategy. 3. Out-of-sample performance. 4. Diversification. 5. Portfolio optimization. I. Campani, Carlos Heitor, orient. II. Título.

Elaborado pelo Sistema de Geração Automática da UFRJ com os dados fornecidos pelo(a) autor(a), sob a responsabilidade de Miguel Romeu Amorim Neto - CRB-7/6283.

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Rio de Janeiro

2021

ACKNOWLEDGMENTS

To CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico), for making this work possible.

To my supervisor, Carlos Heitor Campani, for accepting to help me and guiding me along the way throughout this dissertation.

To Raphael Moses Roquete, who gave his time to help me when I was in need.

To COPPEAD Business School and all professors, who shared their knowledge and allowed me to learn a lot in the past two years.

To my classmates, who shared the path and made the experience even better. I am proud to be part of this cohort.

To my family, that always support me in my journey and made these hard pandemic times slightly easier.

RESUMO

PORTELINHA, Mateus. **Os impactos das criptomoedas na performance de portfólios de ações brasileiras**. Rio de Janeiro, 2021. 37 pp. Dissertação (Mestrado em Administração) – Instituto COPPEAD de Administração, Universidade Federal do Rio de Janeiro, Rio de Janeiro, 2021.

Este estudo analisa os impactos da inclusão de criptomoedas em portfólios de ações brasileiras no período de setembro de 2014 a abril de 2020. As comparações foram realizadas entre portfólios exclusivos de ações contra outros que permitiam ações e criptomoedas. Três portfólios foram usados como benchmarks: a carteira igualmente ponderada e os portfólios tangente e de mínima variância que foram construídos a partir da teoria de média-variância de Markowitz. As performances foram comparadas fora de amostra a partir dos retornos, volatilidades e índices de Sharpe, Sortino e Omega de cada portfólio. Nossos resultados indicam significância estatística positiva para melhorias nos retornos e nas performances ajustadas ao risco após a inclusão das criptomoedas, apesar de também aumentar as volatilidades. A carteira igualmente ponderada com criptomoedas frequentemente superou os portfólios tangente e de mínima variância, que só apresentaram melhores resultados quando mais dados foram incluídos como inputs para os modelos. Além disso, os portfólios que incluíram criptomoedas consistentemente performaram melhor que o IBrX-100 no período estudado. Os resultados deste estudo são importantes para investidores e gestores de fundos, especialmente porque as criptomoedas ainda não são consideradas pela maioria deles.

Palavras-chave: Criptomoedas, Estratégia de portfólio otimizado, Performance fora de amostra, Diversificação, otimização de portfólio, Brasil, Carteira igualmente ponderada, Portfólio de média-variância

ABSTRACT

PORTELINHA, Mateus. **The impacts of cryptocurrencies in the performance of Brazilian stocks' portfolios**. Rio de Janeiro, 2021. 37 pp. Dissertation (Masters Degree in Business Administration) - COPPEAD Graduate School of Business, Federal University of Rio de Janeiro, Rio de Janeiro, 2021.

This study analyses the impact of including cryptocurrencies in Brazilian stocks' portfolios performances from September 2014 to April 2020. The comparisons were made between stocks' only portfolios against portfolios that allowed stocks and cryptocurrencies. Three portfolios served as benchmarks: the naïve but relevant equally weighted portfolio, the tangency and the MVP portfolios built from the Markowitz mean-variance theory. Performances were compared through out-of-sample returns, volatilities, Sharpe, Sortino and Omega ratios. Our results indicate positive statistically significant return and risk-adjusted improvements after the inclusion of cryptocurrencies, although also increasing the volatility. The equally weighted portfolios with cryptocurrencies often outperformed the tangency and minimum variance models, which only exhibited better results when more data was used as input to the models. Moreover, the portfolios that included cryptocurrencies consistently outperformed the IBrX-100 in the period studied. The results of this study are important for investors and fund managers, especially because cryptocurrencies are yet not considered by most of them.

Keywords: Cryptocurrencies, Optimal portfolio strategy, Out-of-sample performance, Diversification, Portfolio optimization, Brazil, Naïve portfolio, Mean-variance portfolio

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1 INTRODUCTION

Nakamoto (2008) created the most popular cryptocurrency so far, Bitcoin, a peer-to-peer digital currency that would allow online payments directly from one party to another, without the need to go through a financial institution. One of the main features is its independency from central authorities that cannot control the supply, which is predetermined and finite, being ultimately deflationary, what he argues would make it a better store of value. On the other hand, this feature also raises concern about the long term survival capacity of the cryptocurrency.

The aim of this study is to explore the performance of stock portfolios with the inclusion of cryptocurrencies in Brazil. There are still very few studies of cryptocurrencies portfolios in emerging markets and therefore this paper seeks to contribute to this literature. The hypothesis is that the inclusion of cryptocurrencies in stock portfolios will improve the out-of-sample performances.

Brazil is a country with its own peculiarities. Government bonds had higher returns than stocks for the period of 2004 to 2016 (Andrino & Leal, 2018), which go against what should be expected by the theory of higher risks and higher returns. On the other hand, Brazil is right now going through an unprecedented reduction of interest rates, with the target that was 14.25% per year in October 2016 being now 2.00% in December 2020 (Brazilian Central Bank, 2020). The real interest rates have also reduced during the period and many investors started looking for investments other than the fixed income market. The Brazilian stock market boosted during 2020, with the entrance of millions of new investors. Cryptocurrencies are getting more and more attention from investors and many of them do consider this asset class in their portfolios. As a natural consequence, we want to understand if cryptocurrencies should be considered as a good option for a diversified stock portfolio.

Bitcoin gained more attention from the public and the media when its prices started skyrocketing. The related literature for cryptocurrencies started then to develop. Currently there are more than 5,000 cryptocurrencies with 150 billion dollars of total market capitalization, with Bitcoin, Ethereum and Ripple being the three biggest ones (CoinMarketCap, 2020).

This new financial instruments have exhibited high returns, but also very high volatility (Elendner, Trimborn, Ong, & Lee, 2017; Hu, Parlour, & Rajan, 2019), leading many investors to not enter the market due to higher risks and also because of uncertainties about the long term sustainability. Cheah and Fry (2015) tested its fundamental value to not being significantly different than zero in the long term, while (Kim, 2017) showed that cryptocurrencies are already reducing transaction costs when used as an intermediary for exchange, which argue in favour of the creation of value by the use of cryptocurrencies.

In order to better understand cryptocurrencies as an asset class, several studies aimed at understanding its potential as a diversifier. The correlations with other established asset classes like global stocks, bonds, gold, commodities, currencies and real estate are very low, close to zero in many cases (Brière, Oosterlinck, & Szafarz, 2015; Elendner et al., 2017; Hu et al., 2019). These finding caught the attention for their potential to be included in portfolios as a good diversifier.

At the same time, cryptocurrencies exhibit high correlations among themselves, especially with Bitcoin, since many of them are traded against this cryptocurrency instead of against fiat currencies. However, there is still evidence of idiosyncratic risks across cryptocurrencies according to the recent literature (Elendner et al., 2017; Hu et al., 2019; Mensi, Rehman, Al-Yahyaee, Al-Jarrah, & Kang, 2019).

Cryptocurrencies showed significant results at increasing the performance of portfolios of stocks and bonds (Platanakis & Urquhart, 2019b) and an emerging market currencies basket (Carrick, 2016). The results are also consistent for three different regions: US, China and Europe (Kajtazi & Moro, 2019). In Brazil, this is the first academic study to analyse this issue.

Three portfolio benchmarks were considered: the tangency and the minimum variance portfolios, built from the Markowitz mean-variance theory (Markowitz, 1952), and the equally weighted portfolio, as in DeMiguel, Garlappi, and Uppal (2009) study. The performance was compared against the Brazilian stock market index IBrX-100 and against CRIX (Trimborn & Härdle, 2018), an index representing the cryptocurrencies market.

The results show that the inclusion of cryptocurrencies in Brazilian stocks' portfolios was successful to increase the out-of-sample returns and the overall risk-adjusted performances, as measured by Sharpe, Sortino and Omega ratios. However, it often increased the volatility, augmenting the risk exposure of investors. Among the portfolio benchmarks, the equally weighted ones were often better than the minimum variance and tangency portfolios, which only exhibited better results when more data were used as input to the model. Furthermore, the portfolios that included cryptocurrencies were consistent to outperform the IBrX-100 index returns and risk-adjusted measures, while they underperformed against the CRIX in the period. However, the CRIX returns exhibited exceedingly high volatilities, being impeditive for almost all investors in practical matters. The paper continues with a literature review, followed by the data and methodology, the results, and the conclusions.

2 LITERATURE REVIEW

2.1 Cryptocurrencies

Despite the name, cryptocurrencies still face many doubts regarding their capacity to function properly as currencies. Glaser, Zimmermann, Haferkorn, Weber and Siering (2014) indicate that Bitcoin's new users mainly use it as an asset class rather than a currency. One of the main challenges is its capacity to store value, since the price volatility is very high. Schilling and Uhlig (2019) created a theoretical model of currency competition to understand the dynamics between Dollar and Bitcoin and showed that Bitcoin must gradually disappear as a mean of exchange.

The lower interference from regulators and central authorities are features that could be seen as an advantage for cryptocurrencies, but also raises concern about the security and the vulnerability to price manipulation. In fact, it was found that suspicious trading from a single actor that occurred in Mt. Gox exchange, the biggest bitcoin exchange at that time (based in Tokyo, Japan) was likely the cause for the increase of Bitcoin's prices in the days they occurred (Gandal, Hamrick, Moore, & Oberman, 2018). This event showed the risky exposure of the market and eventually led to the bankruptcy of Mt. Gox exchange.

To better understand this new asset class, part of the literature is dedicated to describing its main properties and what explain them. For periods ranging from 2013 to 2017, cryptocurrencies mainly exhibit high returns and high volatilities (Elendner et al., 2017; Hu et al., 2019) and the ones with lower market values present higher returns and volatility (Elendner et al., 2017; Phillip, Chan, & Peiris, 2018), sharing the size effect with stocks. The daily realized volatility from 2012 to 2017 also shows a positive relation with the trading volume (Aalborg, Molnár, & de Vries, 2019; Bianchi, 2020). From a Copula-based Granger Causality in Distribution (CGCD) test from 2013 to 2017, Google searches for the term "Bitcoin" seem to have a causal bidirectional relationship with Bitcoin returns in the right and left tail, although it is not present in the central distribution (Dastgir, Demir, Downing, Gozgor, & Lau, 2019). Bouri, Gupta and Roubaud (2019) conduct a rolling window analysis and show that cryptocurrencies present herding behaviour for the period of 2013 to 2018, especially in times of economic policy uncertainty.

One important characteristic that was observed in this asset class is the potential for the formation of bubbles. Bitcoin attracted a lot of speculative money and Cheah and Fry (2015) used an econophysics model to show that in 2013 a crash in December was preceded by a bubble from January to November. Fry and Cheah (2016) find a negative bubble from 2014 for Bitcoin and Ripple. These results contribute for their difficulties of serving as a currency and also raise questions about the long-term sustainability.

Motivated by the apparent lack of intrinsic value that raises doubts for the long term potential of cryptocurrencies, Kim (2017) studied the claim that transaction costs of cryptocurrencies are lower due to its decentralized and deregulated features. Through the study of the transaction costs of buying and selling Bitcoins in different currencies, thus using it at an intermediary for exchange, he found that it was 1.2% to 1.9% lower than the rates of the best foreign exchange rates.

Several studies approach the efficient market hypothesis (EMH), proposed by Malkiel and Fama (1970), for cryptocurrencies. Urquhart (2016) showed that Bitcoin was not weakly efficient for the period of 2010 to 2016, although it started to become more efficient since August of 2013, while Bariviera (2017) found that it was weakly efficient since 2014 for the returns, despite showing persistence for the volatility. Nadarajah and Chu (2017) argued that applying an odd integer power transformation to the returns would make it in line with the EMH without any loss of information. Al-Yahyaee, Mensi and Yoon (2018) compared Bitcoin against gold, currency and global stocks from 2010 to 2017 and found that Bitcoin was significantly more inefficient than the other three markets.

Vidal-Tomás and Ibañez (2018) investigated the presence of semi-strong efficiency, examining if Bitcoin returns behave because of its own events or to monetary policy ones, using data from two exchanges: Mt. Gox, from 2011 to 2014, and Bitstamp, from 2011 to 2017. They show that Bitcoin is semi-strong inefficient regarding monetary policy news, since it does not respond to these events and for Bitcoin events it responds to negative events in the Mt. Gox and Bitstamp markets, and to positive news only in the Bitstamp market.

Cryptocurrencies also present significant spreads for arbitrage. There are relevant deviations in the prices across exchanges, creating the opportunity for profits by investors. These deviations are higher for exchanges from different countries than for exchanges in the

same country. On the other hand, there are significant barriers to this supposed arbitrage opportunity, especially the difficulty for the capital flow between countries (Makarov & Schoar, 2020).

When analysed against other asset classes, cryptocurrencies exhibit another interesting feature – low correlation. This opened up the possibilities for its use as portfolio diversifiers, hedge or even safe haven. The low correlations, in many cases very close to zero, have been tested for several established asses classes, like global stocks, currencies, gold, bonds and real estate (Brière et al., 2015; Elendner et al., 2017; Hu et al., 2019), although it might increase in bearish markets (Klein, Pham Thu, & Walther, 2018).

Dyhrberg (2016) compared several characteristics of Bitcoin to gold, like the finite supply, its independency from authorities and high price volatilities. She argued that, like gold, Bitcoin could be used as a hedge against the Financial Times Stock Exchange (FTSE) Index, while for the dollar it only showed a short-term hedging capacity. In contrast, Klein et al. (2018) defend that it cannot be compared to gold, since they could not find any evidence of a stable hedging capability from Bitcoin, especially because of the way it behaves in distress scenarios, when the correlation with other assets increases.

Trying to understand if cryptocurrencies could act as more than a diversifier, Bouri, Molnár, Azzi, Roubaud and Hagfors (2017) found that Bitcoin is a poor hedge, only presenting relevant safe haven capabilities against shocks in Asian stocks, although it could be used as a good diversifier for many situations. Bouri, Gupta, Tiwari and Roubaud (2017) found that Bitcoin can only be a good hedge against global uncertainty in the short term.

Unlike the comparison to other asset classes, correlations between different cryptocurrencies are high (Corbet, Meegan, Larkin, Lucey, & Yarovaya, 2018; Elendner et al., 2017; Mensi et al., 2019). One possible explanation is that several cryptocurrencies are not traded against a fiat currency, but instead against Bitcoin (Hu et al., 2019). This indicates the existence of idiosyncratic risk across the asset class. Mensi, Rehman, Al-Yahyaee, Al-Jarrah and Kang (2019) showed that the co-movement of cryptocurrencies and Bitcoin varies significantly. It is higher for Dash, moderate for Litecoin, Monero and Ripple and lower for Ethereum. Borri (2019) found that the idiosyncratic risk can be significantly reduced with a cryptocurrency portfolio.

The diversification potential of cryptocurrencies has motivated the literature to better understand the impacts of their inclusion in portfolios.

2.2 Portfolios with cryptocurrencies

Carrick (2016) was able to increase Sharpe and Sortino ratios by including Bitcoin in an emergent market currencies basket, using an equally weighted portfolio from 2011 to 2015. Platanakis and Urquhart (2019b) use data from 2011 to 2018, applying eight popular allocation strategies, and show that including Bitcoin in a stock-bond portfolio increases the out of sample excess return as well as the Sharpe, Sortino and Omega ratios and that it also adds value above and beyond the inclusion of commodities. The findings remain significant even for the period of January 2018 to June 2018, when Bitcoin prices decreased.

Kajtazi and Moro (2019) investigated if the results were similar across three regions: US, China and Europe. They applied the equally weighted and mean-variance portfolios from 2012 to 2017 and found that overall, there was an increase in returns and in volatility, but there are performance improvements in terms of Sortino and Omega ratios. Liu (2019) analysed a number of portfolios consisting of several cryptocurrencies from 2015 to 2018 and concluded that it is possible to have significant diversification gains against any individual asset, by improved Sharpe ratio and utility.

Most of the models for portfolio selection with cryptocurrencies cannot consistently beat the equally weighted portfolio (1/N) (Brauneis & Mestel, 2019; Liu, 2019; Platanakis, Sutcliffe, & Urquhart, 2018). Platanakis and Urquhart (2019a) argue that the high volatility of cryptocurrencies lead to higher potential estimation errors of the parameters and therefore creates difficulties for several models across the portfolio theory, which explains the 1/N portfolio not being consistently beaten. They test the Black-Litterman model with variance-based constraints (VBCs), which is an alternative for dealing with estimations risk: The model was able to achieve better out-of-sample performance than the 1/N model for the cryptocurrency portfolio. Motivated by this literature, we also analyse the 1/N strategy in this study.

Previous authors, therefore, have indicated the potential of cryptocurrencies to improve the overall performance of several different portfolio strategies. Nevertheless,

cryptocurrencies still represent an asset class not much explored by most investors, especially in Brazil. On the other hand, it is true that this asset class is almost a new-born, with short historical data, and needs to mature as time goes by. But the results have been consistent throughout the last years. This boosted literature and increasing interest in the topic was the main inspiration for this work.

3 DATA AND METHODOLOGY

3.1 Data

The data used in the study consist of daily prices from Brazilian stocks and cryptocurrencies during the period of September 2014 until April 2020. September 2014 was chosen as the beginning of the sample because it represents a period that we started to see other cryptocurrencies (besides Bitcoin) becoming more relevant, like XRP and Litecoin.

The cryptocurrencies' daily prices were collected from coinmarketcap.com. Our universe consists of all the ones that have figured as the top five most capitalized coins anytime during the period analysed. Only five cryptocurrencies at each time were picked because it is still a market with very few big players, so the more liquid assets were prioritized. The cryptocurrencies' prices are given in US Dollar, so, in order to convert it to Brazilian currency, we collected the daily exchange rates between the US Dollar and the Brazilian Real. This set of data was collected from Economatica database.

As for the Brazilian stocks, the daily prices were collected from Economatica database, adjusted to cash and stock dividends, stock splits and all related events, and it consists of the ones that have figured in the IBrX-100 index during the period. The index is one of the most important for the performance of stocks in the Brazilian market. It consists of the 100 assets with highest liquidity and which are most representative in the market. The index is rebalanced every four months. We preferred the IBrX-100 instead of the main Brazilian index (Ibovespa) because it has more stocks in its composition, increasing our universe of analysis, and also because the Ibovespa methodology in the past did not fully correspond to market value indexation (Roquete, Leal, & Campani, 2018). Despite the differences, we could observe that the correlation between the IBrX-100 and the Ibovespa is remarkably high (more than 0.97 across the period analysed) and the results would not be qualitatively different.

Some of the calculations in this paper required a risk-free rate. Therefore, we considered the CDI rates, a quite common benchmark for investments in Brazil serving as the opportunity cost with no risk. The daily values of the CDI rate were collected from the Economatica database.

3.2 Estimation procedure

In order to analyse the impact of including cryptocurrencies in portfolios of Brazilian stocks, we used two portfolio strategies: equally weighted portfolios (1/N) and mean-variance portfolios (Markowitz, 1952). The first one was chosen for its simplicity to calculate and execute, being accessible even for individual investors with lower knowledge of more complex models, which also makes it very popular, while still performing better than more complex models in many different situations (Brauneis & Mestel, 2019; Liu, 2019). The second one was chosen due to its relevance, being a common first step for investors who want to get into more complex calculations to optimize their portfolios.

The portfolios were calculated from the universe of assets consisting of stocks listed in the IBrX-100 index in the date of each rebalance, along with the five cryptocurrencies with highest market capitalization at that time, with short positions not being allowed. For each strategy, two portfolios were selected, one picking from all the universe of assets mentioned before and the other being restricted to the stocks. Despite choosing from all of the assets, the portfolios that allow stocks and cryptocurrencies were not obligated to include a minimum weight of cryptocurrencies, leading to situations in which both portfolios were identical until the next rebalancing date. We compared, when cryptocurrencies were available, how often they were present in the optimized portfolios and, if so, how their performances compared to the stocks' only portfolio.

In Brazil, companies can have different types of stocks traded in the exchange market and it is not uncommon to have more than one stock from the same company appearing in the IBrX-100 index. Therefore, to avoid diversification issues, only one stock of the same company was allowed in the final portfolio: the most liquid one. The portfolios were rebalanced in a frequency of four months, matching the schedule of the IBrX-100 index, every beginning of January, May, and September, when the companies that enter and leave the index are released.

We considered four months of data as input for the rebalances. Therefore, the first portfolio was calculated four months after the first data because it was the time when we had enough information to build our portfolios. After selecting the portfolios, the analysis followed an out-of-sample performance estimation process, since an in-sample process would not be

realistic for investors, and it would also assume no estimation errors of the parameters. This process was repeated for all rebalancing periods of four months.

The performance of the portfolios was analysed for the following four months period, until the next rebalancing date. We compared how the portfolios with cryptocurrencies perform against the ones without them, and also how the models performed against each other. The portfolios' performances were also compared against benchmarks as the Brazilian stock market, such as IBrX-100, and CRIX (Trimborn & Härdle, 2018), an index representing the cryptocurrencies market. We were then able to perform the full analysis and better understand the impact of adding cryptocurrencies in portfolios of Brazilian stocks.

3.3 Equally weighted portfolios

The equally weighted, or naïve portfolios, are calculated by simply assigning a weight of 1/N for each of the N assets available in each rebalancing date. It therefore does not require any optimization. Due to its simplicity to calculate and execute, DeMiguel et al. (2009) consider it as the benchmark for any other portfolio selection strategy.

In order to better understand the performance of equally weighted portfolios against other more complex portfolio selection strategies, DeMiguel et al. (2009) investigated its out-of-sample performance against fourteen other optimization models, using data from seven datasets, which vary from US equity market segmentations to global market indexes, with periods from as early as 1963 to as late as 2004. They then compared the performance of the out-of-sample Sharpe ratio (Sharpe, 1966), the certainty-equivalent return of a mean-variance investor and the turnover of each model. Their findings show that none of the models is consistently better than the naïve strategy, indicating that diversification gains compensate for the estimation error of other models.

Since our universe consists of several assets for each period, we had to limit the number of assets in the selected portfolio in order to keep complexity from getting too high. Similar to Leal and Campani (2016), in every rebalancing date the assets were ranked by performance in the previous four months, which was done by the calculation of their Sortino ratio in the period (Sortino & Van der Meer, 1991) (more on this below), and the twenty better performers were selected to the portfolio, each one with a five per cent weight. Leal

and Campani (2016) argue that the amount of twenty assets showcased a great trade-off between diversification and rebalancing costs.

3.4 Markowitz portfolios

In the mean-variance portfolio selection model proposed by Markowitz (1952), investors optimize the trade-off between risk and return of a portfolio. Several efficient portfolios are calculated from the asset pool, each of them maximizing the expected returns for every possible value of expected variance of the returns, known as the efficient frontier. Investors can then select the portfolio with highest expected return for a given level of risk, determined by their own assessment or the level of risk they want to their portfolio.

In our study, two different portfolios were calculated from the mean-variance model: the first was the minimum variance portfolio (MVP), which is the portfolio in the efficient frontier with the lowest variance; the second one was the tangency portfolio, which was selected as the portfolio that maximizes the Sharpe ratio.

Some restrictions were placed on the final weights allowed in the optimization. Each asset from the selected portfolio cannot have a weight of less than 1% or more than 30%. Thomé Neto, Leal and Almeida (2011) argue that the minimum weight restriction avoids the rebalancing costs getting too high and the maximum weight restriction helps to keep an efficient diversification in the portfolio.

3.5 Performance assessment

In order to compare the models, we used a set of metrics. First, we started by computing the daily return R_t of the portfolios, based on the daily returns of the individual assets and their weights in the portfolios. The cryptocurrencies are traded every day, while the stocks follow a 252 calendar, only being traded in working days. To unify the data, we adjusted the cryptocurrencies data to the stocks calendar, by aggregating the returns in non-working days to the following working day.

To present the results, we calculated the annualized geometric returns RG as in equation 1, using the 1,315 days of the sample period, as well as the annualized volatility (proxied by the standard deviation of daily returns), following equation 2.

$$RG = \prod_{t=1}^{1,315}(1 + R_t)^{252/1,315} \quad (1)$$

$$Vol = \sqrt{252 * \left[\frac{1}{1,314}\sum_{t=1}^{1,315}(R_t - \bar{R})^2\right]} \quad (2)$$

Although it might give us a good first assessment, the returns alone are not sufficient to adequately compare portfolios due to different risk levels. Therefore, we used the Sharpe ratio (equation 3), a tool widely used in the literature to compare portfolios with different risk levels, where R_p is the average return of the portfolio, R_f is the average risk-free rate and σ_p is the standard deviation of the portfolio.

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p} \quad (3)$$

The Sortino ratio (equation 4) is similar to the traditional Sharpe ratio, but only considers the standard deviation of the downside returns (in respect to the average return), instead of all of the returns (Sortino & Van der Meer, 1991). The reference return to define a downside is the risk-free rate. Equation 4 presents its formula, in which R_p is the average return of the portfolio, R_f is the average risk-free rate, and σ_d is the standard deviation of the downside portfolio returns (and it considers all other returns as zero).

$$Sortino\ Ratio = \frac{R_p - R_f}{\sigma_d} \quad (4)$$

To complement the Sharpe and Sortino ratios, we also use the Omega ratio (Keating & Shadwick, 2002), pictured in equation 5, where F is the cumulative probability distribution function of the returns and Θ is the target return threshold that will define what is a gain versus a loss: the risk-free rate is again set as this threshold.

$$Omega\ Ratio = \frac{\int_{\Theta}^{\infty}[1 - F(r)]dr}{\int_{-\infty}^{\Theta}F(r)dr} \quad (5)$$

4 RESULTS

4.1 Main results

Table 1 presents the descriptive statistics for the portfolios. For all models, the annualized geometric returns of the stocks+CCs portfolios (i.e., the portfolios that allow cryptocurrencies) were higher than the stocks-only portfolios. Following the bootstrap method (Efron, 1992), we tested the null hypothesis that the difference between the stocks+CCs and stocks-only portfolios returns were equal to zero against the alternative hypothesis that these returns were different than zero. We found statistical significance to reject the null hypothesis for the equally weighted portfolios and for the tangency portfolio. However, it is important to highlight the poor performance of the stocks-only tangency portfolio, presenting lower returns than the IBrX-100 index in the period studied.

Table 1: Descriptive statistics of the portfolios

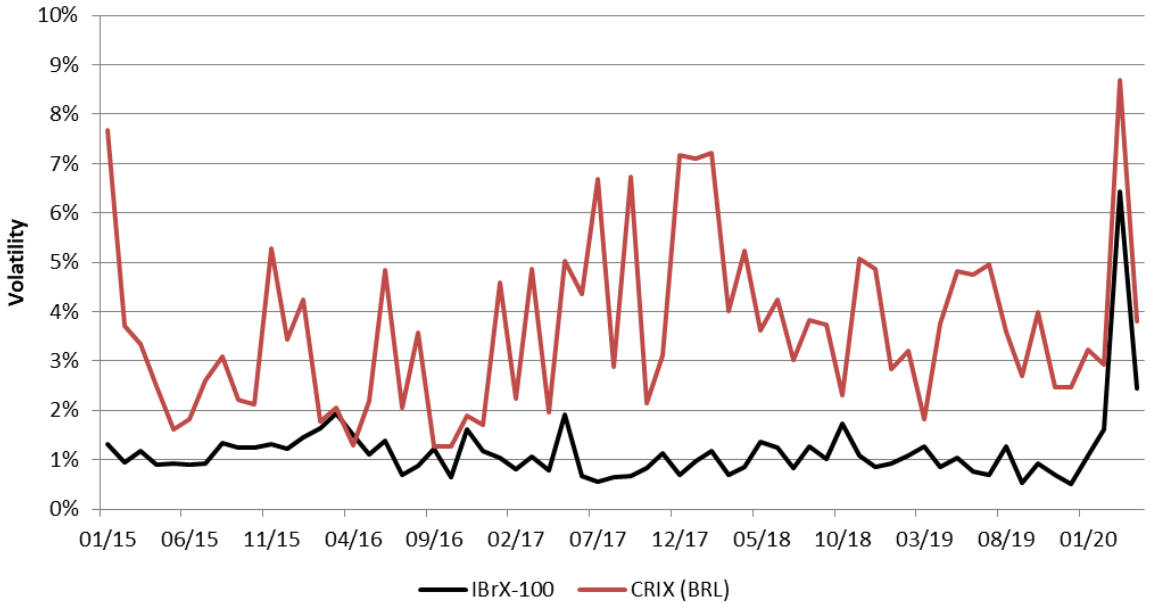
Model/ Index	Portfolio	Annual geom. return (%)	Annual vol. (%)	Sharpe ratio	Sortino ratio	Omega ratio	Average daily return (%)	Median daily return (%)	Minimum daily return (%)	Maximum daily return (%)
Equally Weighted	Stocks-only	12.37	26.29	0.01	0.02	1.12	0.06	0.12	-16.81	11.37
	Stocks+CCs	42.77***	31.82	0.06**	0.09***	1.29**	0.16	0.13	-16.81	15.03
MVP	Stocks-only	10.85	19.35	0.01	0.01	1.14	0.05	0.11	-13.14	10.95
	Stocks+CCs	12.38	19.40	0.01	0.02	1.16	0.05	0.11	-13.14	10.95
Tangency	Stocks-only	4.45	24.98	0.00	-0.01	1.06	0.03	0.10	-17.47	12.87
	Stocks+CCs	13.36*	26.02	0.02*	0.02*	1.13	0.06	0.11	-17.47	12.87
IBrX-100		10.58	26.81	0.01	0.01	1.10	0.05	0.08	-14.89	13.75
CRIX (BRL)		132.32	76.95	0.09	0.13	1.33	0.45	0.40	-37.95	23.99

Note: The statistics refer to the 1,315 daily returns between January 2015 and April 2020. * denotes 10% significance, ** denotes 5% significance and *** denotes 1% significance measured by a bootstrap test. For the portfolios that included cryptocurrencies, the test was performed for the returns and the Sharpe, Sortino and Omega ratios, under the null hypothesis that the difference to the stocks-only portfolio was equal to zero and alternative hypothesis that it was different than zero.

Alongside the higher returns, the stocks+CCs portfolios also exhibited higher volatilities, expressed as the standard deviation. Also, for the equally weighted portfolio, the volatility of the stocks+CCs portfolio was considerably higher than the stocks-only portfolio.

This effect can be explained by the higher volatilities of cryptocurrencies against stocks in the period of analysis, as can be seen in the comparison of the monthly volatilities of the daily returns of the IBrX-100 and CRIX indexes (Figure 1), the first composed of Brazilian stocks and the second of cryptocurrencies.

Figure 1: Daily returns monthly volatility (standard deviation) of the IBrX-100 against the CRIX (BRL)



For the three models, the stocks+CCs portfolios performed better against the stocks-only portfolios in the Sharpe, Sortino and Omega ratios in the period of analysis. The results from the three ratios were statistically significant for the equally weighted portfolio, while for the tangency portfolio only the Sharpe and Sortino ratios were statistically significant and none of them were for the MVP portfolio.

Looking carefully to the information presented in Table 1, we observed that, in some cases, the minimum and maximum daily returns of the stocks+CCs and stocks-only portfolios were equal, suggesting there were periods that the two portfolios were identical and, therefore, the weight of the cryptocurrencies was zero. Table 2 shows, for each model, how many rebalances resulted in a portfolio where the weight of the cryptocurrencies was equal to zero.

Table 2: Rebalances where the weight of cryptocurrencies in the stocks+CCs portfolios was zero

Model	Rebalances without cryptocurrencies
Equally weighted	4
MVP	4
Tangency	3

Note: The data refer to the 16 rebalancing dates in the period of analysis.

Considering the high amount of rebalances that did not include cryptocurrencies in the stocks+CCs portfolios, we decided for an additional test that generated new portfolios with a forced weight for cryptocurrencies. These portfolios were calculated using the equally weighted portfolio selection model, where the W_c is the weight of cryptocurrencies and W_s is the weight of stocks, where W_s = 100% - W_c. Six new portfolios were calculated, with W_c receiving the values of 5%, 10%, 15%, 20%, 25% and 30%. The assets selections were made in two parts: the stocks and the cryptocurrencies. For the stocks, 20 assets were selected following the same methodology applied before, which resulted in the same stocks present in the stocks-only portfolio, each one with weight equal to W_s/20. For the cryptocurrencies, all of the five that were available in each rebalancing date were selected, each one with weight W_c/5. Following the same procedures used previously, Table 3 presents the descriptive statistics for the new portfolios.

Table 3: Descriptive statistics of the equally weighted portfolios that include cryptocurrencies

Portfolio	Annual geom. return (%)	Annual vol. (%)	Sharpe ratio	Sortino ratio	Omega ratio	Average daily return (%)	Median daily return (%)	Minimum daily return (%)	Maximum daily return (%)
Stocks+CCs	42.77***	31.82	0.06**	0.09***	1.29**	0.16	0.13	-16.81	15.03
Stocks 95% + CCs 5%	23.87***	25.95	0.04***	0.05***	1.20***	0.10	0.16	-18.39	11.47
Stocks 90% + CCs 10%	34.86***	27.11	0.06**	0.08***	1.27***	0.13	0.16	-19.86	11.57
Stocks 85% + CCs 15%	45.38***	29.24	0.07***	0.10***	1.32***	0.17	0.14	-21.24	11.67
Stocks 80% + CCs 20%	55.44***	31.96	0.08***	0.12***	1.35***	0.20	0.13	-22.53	13.20
Stocks 75% + CCs 25%	65.06***	35.02	0.08***	0.13***	1.37***	0.22	0.13	-23.74	15.21
Stocks 70% + CCs 30%	74.24***	38.28	0.09***	0.14***	1.38***	0.25	0.13	-24.88	17.00

Note: The statistics refer to the 1,315 daily returns between January 2015 and April 2020. ** denotes 5% significance and *** denotes 1% significance measured by a bootstrap test. The test was performed for the

returns and the Sharpe, Sortino and Omega ratios, under the null hypothesis that the difference to the stocks-only portfolio was equal to zero and alternative hypothesis that it was different than zero.

All the portfolios exhibited significant results for the returns, Sharpe, Sortino and Omega ratios, when compared to the equally weighted stocks-only portfolios, suggesting that these strategies had superior performances in the period of analysis. Also, some of them had more controlled volatilities than the stocks+CCs portfolio, with the stocks 95% + CCs 5%, the stocks 90% + CCs 10% and the stocks 85% + CCs 15% volatilities values being in a similar level to the IBrX-100 index. In this analysis, when compared to the stocks+CCs equally weighted portfolio, the options which included at least 15% of cryptocurrencies performed better in the returns and in the Sharpe, Sortino and Omega ratios. These new portfolios give investors more options to select the strategy that better suits their risk level tolerance.

It can be concluded by the evidence presented that the inclusion of cryptocurrencies in Brazilian stocks' portfolios were successful to increase the out of sample returns and risk-adjusted performance in the period studied. However, it brings together higher risks for the investor, especially when the cryptocurrencies weights get higher, as in the equally weighted stocks+CCs portfolio, and in the portfolios with pre-selected cryptocurrencies weight above or equal to 20%: however, the risk-adjusted performances remained significantly better. Therefore, investors should balance this trade off according to their risk tolerance. Furthermore, the past performances do not guarantee future performances, especially in cases like cryptocurrencies, which are still a new asset class that present very high volatilities. We are not able to say that the observed behaviour will be indeed repeated in the future, as this market becomes more mature. It is also important to note that the cryptocurrencies returns in the period of analysis were very high, which brought an indication in our tests that the more weight that was given to them in a portfolio the better would be the returns, as well as the risk-adjusted performance, despite the higher volatilities.

4.2 Comparison of the models

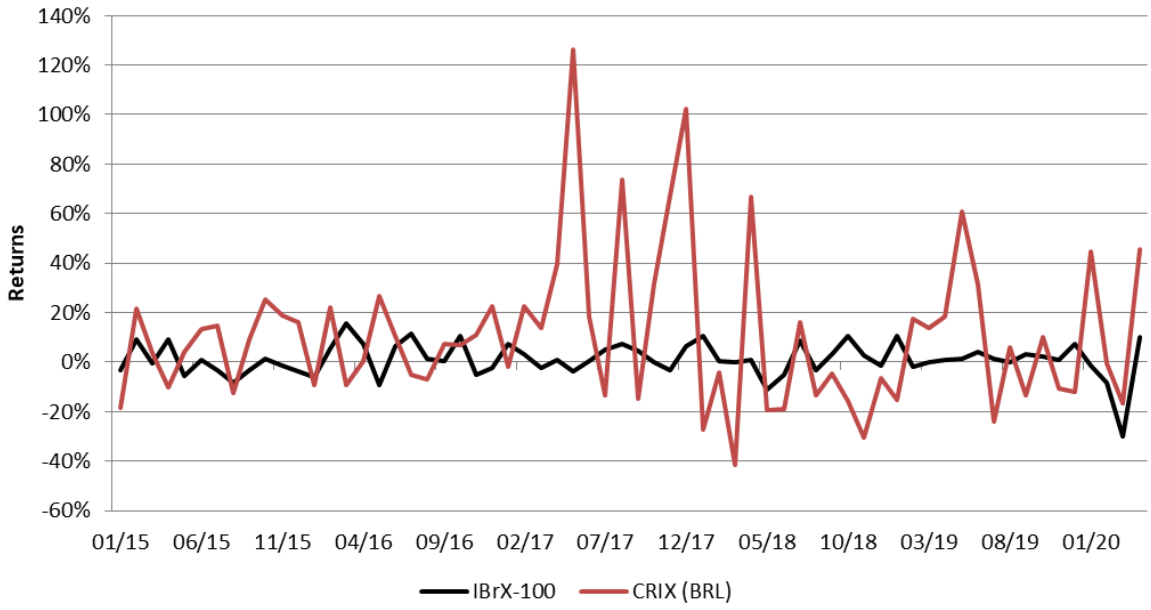
Another aim of this study was to investigate if any particular model of portfolio selection performed better than the others after the inclusion of cryptocurrencies in the portfolios. All seven equally weighted portfolios showed higher returns than the MVP and tangency portfolios. Additionally, these results were statistically significant in all the comparisons, except for the equally weighted stocks 95% + CCs 5% against the tangency

portfolio. From a volatility standpoint, the MVP portfolio presented the lowest value, as expected by construction, followed by the stocks 95% + CCs 5% and the tangency portfolios. The equally weighted stocks+CCs portfolio presented higher volatility in the period, in a similar level to the stocks 80% + CCs 20% portfolio. There was no significant difference between the MVP and tangency portfolios in the returns.

Comparing the Sharpe, Sortino and Omega ratios, all the equally weighted portfolios also performed better than the other models, despite the stocks 95% + CCs 5% not being significant at a 10% level, as well as the Omega of the stocks+CCs and the stocks 90% + CCs 10% against the MVP portfolio. The tangency portfolio presented better Sharpe and Sortino values than the MVP, but a lower Omega, despite not having a statistically significant difference between them.

The explanation for the higher returns and volatility from the equally weighted stocks+CCs portfolio than the tangency and MVP portfolios is due to higher weighting of cryptocurrencies in the period of 2017 to 2018, in which the cryptocurrencies showcased remarkably high returns when compared to the Brazilian stocks (Figure 2), which was followed by also higher volatilities (Figure 1).

Figure 2: Monthly returns of the IBrX-100 against the CRIX (BRL)



The results show that the equally weighted portfolios in general performed better than the MVP and the tangency portfolios. Inside the equally weighted options, the investors could

choose between the different portfolios depending on their risk tolerance levels. The MVP portfolio could still be an option, as it was the portfolio with overall lowest volatility.

4.3 Comparison with indexes

When comparing the returns of the portfolios from each model that include cryptocurrencies against the IBrX-100 index, all of them presented higher values for the period studied, although only the equally weighted portfolios presented a significant result at a 10% significance level. On the other hand, the returns from CRIX were much higher than all the models.

When comparing the volatility of the models against the IBrX-100 index, the tangency portfolio and the equally weighted with pre-defined weight for cryptocurrencies until 15% presented a volatility of similar level to the index, while it was lower for the MVP portfolio and higher for the other portfolios. As for the returns, the volatility of the CRIX index was much higher than all the models.

For the Sharpe, Sortino and Omega ratios, all models performed better than the IBrX-100 index in the studied period, although the results were only significant for the equally weighted portfolios. However, the CRIX index performed better in these metrics than almost all models, except against some of the equally weighted portfolios that forced higher weights for cryptocurrencies.

Therefore, the inclusion of cryptocurrencies in Brazilian stocks' portfolios had a positive impact against a passive strategy of investing in the IBrX-100 index in the period studied. However, as mentioned previously, the investor should be aware of the risks in the inclusion of very volatile assets, like cryptocurrencies, as well as the fact that its behaviour is still unclear since it is a very new asset class. Moreover, the strategy of being fully invested in cryptocurrencies had great results for returns, Sharpe, Sortino and Omega ratios in the period, but with an immense risk and, therefore, should be avoided by almost all of the investors.

4.4 Robustness test: monthly rebalances

To add a robustness check of the results, the rebalances were changed from every four months to monthly. Two major changes are expected: first, the weight of each asset should not move as far from the original value at the rebalancing date, since the next rebalance will

happen more closely; second, the assets in the portfolios will change, since new inputs will be used in the calculation, and also because changes could happen from one month to the other in the selection universe of cryptocurrencies, since the five with most market capitalization could be different. Besides the rebalancing frequency, the previously used procedures were followed, leading to the results presented in Table 4.

Table 4: Descriptive statistics of the portfolios with monthly rebalances

Model/ Index	Portfolio	Annual geom. return (%)	Annual vol. (%)	Sharpe ratio	Sortino ratio	Omega ratio	Average daily return (%)	Median daily return (%)	Minimum daily return (%)	Maximum daily return (%)
Equally Weighted	Stocks-only	13.95	26.40	0.02	0.02	1.13	0.07	0.10	-17.60	10.40
	Stocks+ CCs	28.23*	28.80	0.04*	0.06*	1.22	0.12	0.07	-18.69	10.49
	Stock 95% + CCs 5%	21.70***	25.82	0.04***	0.05***	1.20***	0.09	0.11	-18.67	10.50
	Stock 90% + CCs 10%	29.23***	26.19	0.05***	0.07***	1.25***	0.12	0.13	-19.72	10.52
	Stock 85% + CCs 15%	36.51***	27.41	0.06***	0.08***	1.29***	0.14	0.12	-20.77	10.75
	Stock 80% + CCs 20%	43.52***	29.32	0.07***	0.10***	1.31**	0.16	0.14	-21.80	13.27
	Stock 75% + CCs 25%	50.24***	31.74	0.07**	0.11**	1.33**	0.19	0.12	-22.82	15.58
	Stock 70% + CCs 30%	56.65***	34.55	0.08**	0.12**	1.34**	0.21	0.13	-23.83	17.73
MVP	Stocks-only	10.26	16.08	0.01	0.01	1.14	0.04	0.07	-9.16	5.64
	Stocks+ CCs	12.56	15.98	0.02	0.02	1.17	0.05	0.09	-10.47	5.65
Tangency	Stocks-only	8.18	23.98	0.00	0.01	1.10	0.04	0.13	-18.11	9.56
	Stocks+ CCs	15.36	25.43	0.02	0.03	1.15	0.07	0.09	-18.92	9.66

Note: The statistics refer to the 1,315 daily returns between January 2015 and April 2020. * denotes 10% significance, ** denotes 5% significance and *** denotes 1% significance measured by a bootstrap test. For the portfolios that included cryptocurrencies, the test was performed for the returns and the Sharpe, Sortino and Omega ratios, under the null hypothesis that the difference to the stocks-only portfolio was equal to zero and alternative hypothesis that it was different than zero.

The results from this robustness test lean to similar conclusions as before. However, few changes were spotted, and will be now approached. First, the statistical significances from the equally weighted portfolios returns, Sharpe, Sortino and Omega ratios were slightly reduced, but the only one that lost the significance at a 10% level was the stocks+CCs portfolio's Omega ratio. Moreover, most portfolios that included cryptocurrencies exhibited lower returns and lower volatilities, but with a slightly worst risk-adjusted performance. The

hypothesis is that the more frequent rebalances avoided the cryptocurrencies weights from getting too high in periods when their prices were skyrocketing, as in 2017 and 2018 (Figure 2).

The tangency portfolio also lost the significance of the returns and the Sharpe ratio, despite having an overall better performance. Therefore, it leads us to assume that it happened because of an improvement on the performance from the stocks-only portfolio, which remained as an inferior option when compared to the IBrX-100 index.

To sum up, cryptocurrencies still remained as a good option for investors in order to improve the performances of Brazilian stocks portfolios. But in the period analysed, buy-and-hold 4-month strategies with cryptocurrencies performed better than buy-and-hold 1-month similar strategies. Along with the warnings mentioned previously, it is also important to note that the more frequent rebalances will increase the transaction costs. We did not consider transaction costs for clarity purposes and also because they have been quickly declining with the advances in the Brazilian market, so to apply a value for each transaction could quickly make this study out-dated.

4.5 Robustness test: one-year inputs

This second robustness check brought the rebalances back to every four months and changed the data used as inputs in the portfolio selection models: instead of the previous four months, we now use the previous year data as inputs to the models. This will make the portfolios less reactive to short terms events. The period of analysis will also change, as we need more data to calculate the first portfolios. Therefore, it began in September 2015, instead of January 2015. The results are exhibited in Table 5.

For the equally weighted portfolios, all the options that include cryptocurrencies remain with higher returns than the stocks-only portfolio, as well as higher risk-adjusted performance, and all of these results being statistically significant. Compared to the basis scenario, the portfolios in this test have shown slightly better returns, even if the comparison only consider the same period in both cases, with higher volatilities, but the risk-adjusted performances are also better.

Table 5: Descriptive statistics of the portfolios using yearly data as inputs

Model/ Index	Portfolio	Annual geom. return (%)	Annual vol. (%)	Sharpe ratio	Sortino ratio	Omega ratio	Average daily return (%)	Median daily return (%)	Minimum daily return (%)	Maximum daily return (%)
Equally Weighted	Stocks-only	19.51	27.46	0.03	0.04	1.17	0.09	0.15	-16.21	12.49
	Stocks+ CCs	58.56***	36.62	0.08**	0.12**	1.35**	0.21	0.19	-16.21	22.22
	Stock 95% + CCs 5%	32.05***	27.35	0.05***	0.07***	1.26***	0.13	0.17	-17.79	12.51
	Stock 90% + CCs 10%	43.72***	28.86	0.07***	0.10***	1.20***	0.16	0.20	-19.26	12.54
	Stock 85% + CCs 15%	54.62***	31.30	0.08**	0.12***	1.37**	0.19	0.17	-20.65	13.81
	Stock 80% + CCs 20%	64.83***	34.26	0.09**	0.13***	1.40**	0.22	0.17	-21.96	17.98
	Stock 75% + CCs 25%	74.40***	37.51	0.09**	0.14**	1.42**	0.25	0.16	-23.20	22.02
	Stock 70% + CCs 30%	83.36***	40.90	0.09**	0.15**	1.43**	0.27	0.15	-24.36	25.92
MVP	Stocks-only	8.74	17.85	0.00	0.01	1.13	0.04	0.07	-11.70	9.43
	Stocks+ CCs	17.98***	18.33	0.03***	0.04***	1.24***	0.07	0.08	-13.50	9.50
Tangency	Stocks-only	10.50	24.00	0.01	0.01	1.12	0.05	0.10	-14.14	12.32
	Stocks+ CCs	45.47***	28.49	0.07***	0.11***	1.35***	0.17	0.15	-16.31	16.08
IBrX-100		13.06	27.58	0.02	0.02	1.12	0.06	0.09	-14.89	13.75
CRIX (BRL)		153.40	78.40	0.09	0.14	1.35	0.49	0.43	-37.95	23.99

Note: The statistics refer to the 1,147 daily returns between September 2015 and April 2020. ** denotes 5% significance and *** denotes 1% significance measured by a bootstrap test. For the portfolios that included cryptocurrencies, the test was performed for the returns and the Sharpe, Sortino and Omega ratios, under the null hypothesis that the difference to the stocks-only portfolio was equal to zero and alternative hypothesis that it was different than zero.

However, the major impacts occur in the mean-variance portfolios. Both the MVP and tangency stocks+CCs portfolios exhibited strong improvements in the returns and risk-adjusted performance, without relevant impacts in their volatilities. In both cases, the returns, Sharpe, Sortino and Omega were significantly better than the stocks-only portfolio.

Consequently, the comparison between the models indicates that the equally weighted portfolios are only consistently better in the returns against the tangency portfolio in the scenarios where the weight of cryptocurrencies is forced to at least 25%, and even those cases were not significantly better in the risk-adjusted performance. When compared to the

MVP portfolio, only the case with the weight of cryptocurrencies forced to 5% did not show statistically significant difference in the risk-adjusted performance.

Compared to the IBrX-100 index, the differences between the returns and the Sharpe, Sortino and Omega ratios became statistically significant for the tangency portfolio, while it remained not significant for the MVP portfolio. The hypothesis for the improvements in the MVP and tangency portfolios performances is the following: with more data as inputs, the mean-variance models improved the estimators and, therefore, generated better optimized portfolios.

5 CONCLUSIONS

This paper analysed portfolios using a dataset from September of 2014 until April of 2020, with the objective of understanding the impact of including cryptocurrencies in Brazilian stocks' portfolios. We compared the changes in the returns, volatilities, and Sharpe, Sortino and Omega ratios. We worked with three different portfolio selection models: the equally weighted portfolio, the minimum variance portfolio, and the tangency Markowitz portfolio, each one with and without cryptocurrencies.

The portfolios with cryptocurrencies presented higher total returns for the period analysed than the stock-only portfolios, being statistically significant at a 10% level for the equally weighted portfolios and the tangency portfolio. Notwithstanding, this result comes with an increase in the portfolio volatilities, due to the high-risk characteristic of the cryptocurrencies (highly volatile in the period). Despite the increased volatility, the portfolios with cryptocurrencies performed better in the Sharpe, Sortino and Omega ratios. Therefore, the analysis carried out by this study suggests that cryptocurrencies could be included in stocks' portfolios by Brazilian investors, although with some caution, especially by investors less inclined to riskier investments.

When comparing the different models, the equally weighted portfolios performed better, with higher returns and higher Sharpe, Sortino and Omega ratios, especially in the portfolios with a pre-defined cryptocurrencies weight, which gave investors options to choose from depending on their risk-tolerance: more cryptocurrencies in the portfolio will tend to result in higher expected returns and higher volatilities, with an overall higher risk-adjusted performance. The tangency and minimum variance portfolios only become good alternatives if the data used as inputs for the portfolio selection is extended, since they provided better significant out-of-sample returns, Sharpe, Sortino and Omega in the test using one year of data.

Overall, the strategy of including cryptocurrencies in the portfolios presented consistent results against the IBrX-100 index for both returns and risk-adjusted performances in the period studied. Even regarding the volatility, the investors have options that improve the returns with similar risk to the index.

This study must be considered under two important limitations. One is the short period of data available for cryptocurrencies, which should be addressed as this asset class becomes more mature. Nonetheless, due to the strong gains these assets had in the period studied, the results leaned in the direction that the more cryptocurrencies investors include in their portfolios the better, which might not hold true in the future. Second, transaction costs and income taxes were not considered by this work: we opted to do so for clarity purposes and also because the Brazilian market is evolving very fast with declining transaction costs. The choice of how to quantify transaction costs could impact our results and quickly make them out-dated.

As suggestion for future studies, different portfolio selection models could be applied, with more complex calculations, which could present better returns than the ones analysed in this study, although being suited only for investors with more knowledge. Also, more data could be used as inputs for the models, which was a limitation due to the small database the cryptocurrencies have in the time of this study. Another suggestion would be to analyse the inclusion of cryptocurrencies in portfolios with more asset classes, like fixed income, real estate, currencies, global stocks, and even commodities.

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