Product, operation, and demand relationships between manufacturers and retailers

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A R T I C L E   I N F O

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A B S T R A C T

This paper presents a framework for identifying the most adequate distribution structure between manufacturing and retailers, considering different characteristics of product, operation, and demand. Based on the random generation of different scenarios, it is indicated the adequacy of direct, echeloned, and mixed distribution structures to a given set of demand uncertainty, lead time variability, holding costs, and transportation costs. Sensitivity analyses are also performed to address managerial issues, such as the mission of a Local DC and the convenience of lead time demand pooling. Findings suggest an increase of mixed distribution structures to the detriment of purely direct/echeloned ones.

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1. Introduction

The last three decades have witnessed sharp changes in supply relationships between manufacturers and retailers. Reports on these new supply relationships have already appeared in scholarly literature (Christopher and Peck, 2004; Christopher, 1998, 2000). Several manufacturers have restructured their distribution networks to meet the continuing demand of retailers for smaller stocks and higher service levels (Claassen et al., 2008; Hoek, 1998a, 1998b). The managerial initiatives that culminated in the restructuring of such operations are referred to using a variety of different names: Efficient Consumer Response (ECR), Quick Response (QR), Vendor Managed Inventory (VMI), Continuous Replenishment (CR), and Continuous Replenishment Program (CRP) (Rodriguez et al., 2008; Khadar, 2007; Holmstrom et al., 2002; Helms et al., 2000; Vergin and Barr, 1999; Waller and Johnson, 1999; Ellinger and Taylor, 1999; Liz, 1999; Kiely, 1998; Mathews, 1997; Fiorito and May, 1995; Andraski, 1994).

In general, these initiatives are based on a common goal: to reduce demand uncertainty for the manufacturer and reduce inventory levels at retail combined with a simultaneous increase in service level (Blankley, 2008; Blankley et al., 2008; Collin and Lorenzin, 2006; Lee and Padmanabhan, 1997). According to Vergin and Barr (1999), cooperation and sharing of real time demand information would enable this goal to be achieved, with electronic commerce playing a key role (Affonso et al., 2008; Bharadwaj and Matsuno, 2006; Kirche et al., 2005; Kiely, 1998). However, the literature is inclusive with respect to the homogeneity of results obtained within manufacturers and retailers (Wanke et al., 2008); although the restructuring of distribution by manufacturers has enabled a reduction of inventories at retailers, the benefit is not so clear in the case of manufacturers (Sterman, 2000; Romero, 1991; Harrison and Voss, 1990).

It is possible that the benefits expected with respect to inventory levels and total costs within manufacturers depend on the adherence between their distribution network and the characteristics of the product, demand, and the market service operations (Seifert et al., 2006a, 2006b; Johnson and Stice, 1993; Jones, 1991).

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Wanke and Zinn (2004) provide an example of this based on the VMI initiative, which can be used to illustrate the direct distribution/echeloned distribution under different trade-offs between inventory coverage levels at manufacturers and delivery lead time to retailers. In VMI, the manufacturer’s inventory can be located in an advanced distribution center (DC), and delivery to the retailer can be practically instantaneous. The key question is how the manufacturer should service the retailer. The choices are a centralized inventory at the manufacturer combined with a longer delivery lead time for the retailer (and higher inventory coverage level) or a decentralized inventory, close to the retailer, with faster delivery (and lower inventory coverage level). In essence, the objective consists of knowing whether this managerial initiative should be structured via direct distribution (centralized inventory at the manufacturer or at its single distribution center) or via echeloned distribution (decentralized inventory in a local distribution center).

Qualitatively, the impact of the distribution structure on manufacturers and retailers is relatively well documented (Wanke et al., 2008; Chen et al., 2008; Su, 2007; Kumar and Ruan, 2006), and there is plenty of empirical evidence on the main effects of direct distribution and echeloned distribution (Evers, 1999a; Leeuw and Goor, 1999; Evers and Beier, 1998; Tallon, 1993; Amstel and Amstel, 1985). Indeed, some of the impacts have been understood for over 20 years. For example, Bowersox et al. (1980) argue that echeloned distribution imply higher levels of inventory for the manufacturer and is preferable when the items have low inventory holding costs and there is the possibility of consolidating transport between the factory and distribution center (Croxton and Zinn, 2005; Jayaraman, 1998; Carter and Ferrin, 1996; Mahmoud, 1992). Direct distribution from the manufacturer tends to be seen for items with high inventory holding costs, especially if volumes are high and retailing is close by Bowersox and Closs (1996). The high cost of holding inventories may also inhibit intermediaries interested in keeping them, resulting in the manufacturer using direct distribution to the end customer (Lambert et al., 1998).

According to Levy and Weitz (1998), the choice of distribution structure on the part of the retailer should consider both the total cost associated with each alternative and service to the end customer, i.e., having the product available in store when the customer wants to buy it. Echeloned distribution allows the retailer to operate with fewer inventories, the result of more frequent deliveries from the distribution center. Furthermore, a better balance between surplus and shortages could result from a review, whenever necessary, of the quantities ordered to the distribution center (Berman and Evans, 1998). Direct distribution, because it is time-consuming for the retailer in terms of receiving and processing orders, can lead to less frequent replenishments and shipment consolidation. According to Levy and Weitz (1998), direct distribution to retail is also driven by geographical proximity.

It can be seen that from the perspectives of manufacturers and retailers, the choice of distribution structure is irrelevant when considering the criteria origin/destination distance and purchase volume: greater distances and lower volumes, echeloned distribution with distribution center consolidation; shorter distances and larger volumes, manufacturer–retailer direct distribution. When the criterion of analysis is the level of inventory at the manufacturer and at the retailer, echeloned distribution implies higher inventory levels for the former and lower inventory levels for the latter. With direct distribution, the converse is true.

There is, however, a growing literature on the integration of direct and echeloned distribution structures, as reviewed by Tsay and Agrawal (2004) and Cattani et al. (2004). Most papers are focused on studying competition in price and/or marketing effort (Bell et al., 2002; Cattani et al., 2006). More recently, Kumar and Ruan (2006) examined the strategic forces that may influence the manufacturer’s decision to complement the echeloned distribution with a direct online channel. Su (2007) discussed several research opportunities for supply chain and logistics researches, encompassing business and technology issues.

According to Chen et al. (2008), although most research in this area assumes deterministic demands and ignores the effect of inventories, they incorporated the effects of stochastic demands into a detailed customer distribution choice model in which the demand faced in each distribution structure depends upon their respective service levels. Seifert et al. (2006a, 2006b) departed from the base-stock levels and expected costs for a single facility (Porteus, 1990) and expanded them to several locations, analyzing a situation wherein both the retailers and the manufacturers, while continuing to supply the retailer, have market access, differing from the previous existing literature with the exception of Tsay and Agrawal (1999).

This research differs from previous studies not only for considering a third alternative besides direct and echeloned distribution structures (here called mixed distribution structure) but also for developing a framework, based on inventory portfolio and consolidation effects, for evaluating the adequacy of these three different distribution structures in light of several product, lead time, and demand characteristics (Wanke, 2009; Wanke and Saliby, 2009). The research question to pose now, from the perspective of the manufacturer, is under what conditions can direct and echeloned distribution be used simultaneously to serve the market and in what proportions. More specifically, the main variables that impact these proportions must be assessed – and these proportions determined – in order to minimize the total costs of inventory and transportation in the manufacturer’s distribution network.

The answer to these questions will take as a starting point a network composed of a factory, a Central DC, m Local DCs, and n markets. In these markets, the available distribution policy is a fairly common one (Wanke and Zinn, 2004): receiving CIF directly from the Central DC with a greater lead time or FOB pick up from the Local DC with a shorter lead time. The acronym CIF stands for Cost, Insurance and Freight, usually indicating a sales contract term where the distribution costs occur at the expenses of the manufacturer. On the other hand, the acronym FOB stands for Free on Board. If the contract term is FOB
at manufacturer, the retailer pays for the transportation costs and bears the risk of loss during transit. That is, the manufacturer completes its performance when delivering the conforming products to the carrier at the Local DC.

The paper starts off by modelling the network depicted in Fig. 1. Then, the particular case where \( n = m = 1 \) is analyzed in further detail, thereby allowing the derivation of several analytical expressions. Sensitivity analysis and numerical simulations are performed to address the adequacy of these distribution structures to different operating characteristics. Lastly, managerial implications are addressed.

2. Network modelling

2.1. General case

Let the following variables be relative to a logistics network with \( n \) markets, \( m \) Local DCs, and just one Central DC served by a factory:

- \( D_i \): mean demand at market \( i \), in units/day
- \( s_{D,i} \): standard deviation of demand at market \( i \), in units/day
- \( \rho_{ij} \): correlation between market demands \( i \) and \( j \) met by Local DCs \( l \)
- \( \rho_{ij} \): correlation between replenishment orders generated at Local DCs \( l \) and \( l' \), both serviced by the Central DC
- \( \rho_{ij} \): correlation between market demand \( i \) and replenishment order generated at Local DC \( l \), both serviced by the Central DC
- \( W_{il} \): percentage of market demand \( i \) serviced via Local DC \( l \)
- \( W_{il} \): percentage of market demand \( i \) serviced via Central DC, noting that \( \sum_{l=1}^{m} W_{il} + W_{il} = 1 \) for all \( i \)
- \( IC_i \): inventory coverage level to be kept at the Local DC \( l \), in consumption days
- \( C_{el} \): transference cost from Central DC to Local DC \( l \), in $/unit
- \( C_{di} \): direct distribution cost from Central DC to market \( i \), in $/unit
- \( h \): inventory holding cost, in $/unit/day
- \( k_N \): safety factor at \( l \) Local DCs, assuming Normal distribution
- \( k_C \): safety factor at Central DC, assuming Chebychev’s inequality (Appendix A)
- \( LT_l \): mean lead time from Central DC to Local DC \( l \), in days
- \( SLT_{ij} \): standard deviation of lead time from Central DC to Local DC \( l \), in days
- \( LT_{il} \): mean lead time from factory to Central DC, in days
- \( SLT_{il} \): standard deviation of lead time from factory to Central DC, in days.

Fig. 1. Direct vs. echeloned distribution.
Further, let the mean \((E_A)\) and variance \((V_A)\) of the aggregate market demand be given by:

\[
E_A = \sum_{i=1}^{n} D_i, \tag{1}
\]

and

\[
V_A = \sum_{i=1}^{n} s_{D_i}^2 + 2 \left( \sum_{i=1}^{n} \left( \sum_{j=1}^{i-1} \rho_{ij} s_{D_j} s_{D_j} \right) \right). \tag{2}
\]

According to Wanke (2009, 2010), the market demand allocated to a given Local DC \(l\) presents mean \((E_{D,l})\) and variance \((V_{D,l})\) given as follows:

\[
E_{D,l} = \sum_{i=1}^{n} W_{il} D_i, \tag{3}
\]

and

\[
V_{D,l} = \sum_{i=1}^{n} W_{il}^2 s_{D_i}^2 + 2 \left( \sum_{i=1}^{n} \left( \sum_{j=1}^{i-1} W_{ij} W_{il} \rho_{ij} s_{D_j} s_{D_j} \right) \right). \tag{4}
\]

Now, the mean \((E_{D,C})\) and the variance \((V_{D,C})\) of the market demand allocated to the Central DC, resulting from a direct distribution structure to \(i\) markets is given next:

\[
E_{D,C} = \sum_{i=1}^{n} W_{iC} D_i, \tag{5}
\]

and

\[
V_{D,C} = \sum_{i=1}^{n} W_{iC}^2 s_{D_i}^2 + 2 \left( \sum_{i=1}^{n} \left( \sum_{j=1}^{i-1} W_{iC} W_{ij} \rho_{ij} s_{D_j} s_{D_j} \right) \right). \tag{6}
\]

The mean replenishment orders allocated to the Central DC, generated by serving a given Local DC \(l\) (echeloned distribution structure), is:

\[
E_{R,C,l} = \sum_{i=1}^{n} W_{il} D_i. \tag{7}
\]

However, the variance of the replenishment orders \((V_{R,C,l})\) allocated to the Central DC, generated by the requests of a given Local DC \(l\), can be determined based on \(Q_l\) (Appendix B), that is, the replenishment order quantity - or lot size - from Local DC \(l\) to Central DC:

\[
V_{R,C,l} = \left( \sum_{i=1}^{n} W_{il} D_i \right) Q_l = \frac{\left( \sum_{i=1}^{n} W_{il} D_i \right)}{C_1} Q_l, \tag{8}
\]

where the relation between \(Q_l\) and \(IC_l\) is given by:

\[
Q_l = E_{DJ} \cdot IC_l, IC_l > 1. \tag{9}
\]

Based on previous results, the aggregate mean \((E_C)\) and the variance \((V_C)\) of the market demands and the replenishment orders allocated to the Central DC – from both direct and echeloned distribution structures – can easily be determined. The mean is simply:

\[
E_C = \sum_{i=1}^{m} E_{R,C,l} + E_{D,C}. \tag{10}
\]

As regards the variance, for the sake of simplicity, it is assumed that both the correlations between replenishment orders generated at different Local DCs \(l\) and the correlations between market demands \(i\) and replenishment orders generated at Local DCs \(l\) are equal to zero. More precisely, \(\rho_{l,l'} = 0\) for all \(l\) and \(l'\), \(l \neq l'\), and \(\rho_{i,j} = 0\), for all \(i\) and \(l\). This assumption is quite reasonable (Seifert et al., 2006a), unless these replenishment orders are perfectly synchronized and occur at constant time intervals. Thus:

\[
V_C = \sum_{l=1}^{m} V_{R,C,l} + V_{D,C}. \tag{11}
\]

The network safety stock \((SS)\), assuming that the service factor \(k_N\) is equal for all \(l\) Local DCs, is given by:
\[
SS = k_c \sqrt{E_{c,t}^2 s_{t,c}^2 + V_c LT_c} + \sum_{i=1}^{m} k_n \sqrt{E_{d,i}^2 s_{i,t}^2 + V_D LT_i}.
\]

Whereas the total cost (TC) of holding safety stocks and of direct/echeloned distribution to \( n \) markets is given next:

\[
TC = h \left( k_c \sqrt{E_{c,t}^2 s_{t,c}^2 + V_c LT_c} + \sum_{i=1}^{m} k_n \sqrt{E_{d,i}^2 s_{i,t}^2 + V_D LT_i} \right) + \left( \sum_{i=1}^{n} W_{i,c} D_i C_{d,i} \right) + \left( \sum_{i=1}^{m} C_e E_{d,i} \right).
\]

2.2. Particular case

The impact of different distribution structures on network safety stock levels and total costs can be generalized considering the particular case where \( n = m = 1 \) (cf. Fig. 2), that is, a single market served either by a single Local DC or by the Central DC. Let also \( W_{1,c} = 1 - W_{1,l} \), \( LT_c = x LT_1 \), \( s_{i,t} = y s_{i,t,1} \), and \( C_{d,1} = z C_{d,1} \), where \( x \) is the ratio between lead time means, from the factory to the Central DC and from the Central DC to the Local DC; \( y \) is the respective lead time standard deviation ratio; and \( z \) is the ratio between distribution and transference costs, from the Central DC to the market and from the Central DC to the Local DC. Hence, it follows that:

\[
SS = k_c \sqrt{D_{1}^2 s_{1,c}^2 + (W_{1,1} D_{1}^2 (IC_1 - 1) + (1 - W_{1,1}) s_{D,1}^2) x LT_1} + k_n W_{1,1} \sqrt{D_{1}^2 s_{1,t}^2 + s_{D,1}^2 LT_1},
\]

and

\[
TC = h SS + D_1 C_{d,1} (1 - W_{1,1}) y + W_{1,1}.
\]

Differentiating \( CT \) with respect to \( W_{1,1} \) and equaling it to zero, the value of \( W_{1,1} \) that minimizes total costs and, therefore, characterizes the optimal distribution structure, is obtained:

\[
W_{opt} = - \left( \frac{a_1}{a_2} + \frac{1}{2} \right) \left( \frac{(a_1 a_7 + a_8) ((2 a_3 a_6 - a_2 a_5) (a_1 a_7 + a_8) + a_2 (a_2^2 - 4 a_2 a_6) (a_1 a_7 + a_8)^2 + 4 a_1^2 (-a_2 a_3 a_5 + a_5 a_2^2 + a_2^2 a_4))}{(a_2^2 a_2^2 - 6 a_2 (a_1 a_7 + a_8)^2)} \right),
\]

where \( a_1 \) to \( a_8 \) are just support variables, determined during the differentiation process, used in order to make this presentation more clear. Support variables are as follows:

\[
a_1 = \frac{1}{2} h,
\]

\[
a_2 = 2 k_c x LT_1 (D_{1}^2 IC_1 - D_1^2 + s_{D,1}^2),
\]

\[
a_3 = -2 k_c x LT_1 s_{D,1},
\]

\[
a_4 = D_{1}^2 y^2 s_{1,t}^2 + x LT_1 s_{D,1}^2,
\]

\[
a_5 = -2 x LT_1 s_{D,1}^2,
\]

\[
\text{One single factory} \quad \text{One Central DC} \quad \text{One Local DC} \quad \text{One market}
\]

\[\text{Echeloned distribution via Local DC to markets}\]

\[\text{Direct distribution from Central DC to markets}\]

\[\text{Fig. 2. Particular case where} \ n = m = 1.\]
\[ a_6 = xLT_1 \left( D_1^1IC_1 - D_1^2 + s_{D_1}^2 \right). \]  
\[ a_7 = \frac{k_N \left( 2D_1^2s_{LT_1}^2 + 2s_{D_1}^2LT_1 \right)}{\sqrt{D_1^2s_{LT_1}^2 + s_{D_1}^2LT_1}}, \]  
and
\[ a_8 = D_1C_1 \left( 1 - z \right). \]

If \( W_{opt} = 0 \), a direct distribution structure is preferable to the detriment of an echeloned one. The reverse is true for \( W_{opt} = 1 \). A mixed distribution structure prevails whenever \( W_{opt} \) falls within values greater than 0 and smaller than 1. In this case, both direct and echeloned distributions coexist simultaneously in percentages that add up to 1 (cf. Fig. 3).

Before proceeding, it is worth commenting on how large this network structure can be handled in practice by optimization softwares. Although an analytic expression can only be derived for the particular case where \( n = m = 1 \), Wanke et al. (2009) showed that the LGO solver system (from Pinter Consulting Services, available in AIMMS 3.9) can be successfully employed to determine optimal solutions for similar network problems – that is, containing inventory portfolios and consolidation effects formulae in their objective functions – up to \( n = m = 5 \). This solver system is also available in Maple 12 and it is called as the GlobalSolve procedure.

3. Sensitivity analyses

Sensitivity analyses were conducted via analytical modelling and simulation in order to quantify the relative impact of several variables on the optimal distribution structure.

More precisely, sensitivity analyses were first conducted on Eqs. (14)–(16) for different levels of lead time ratios \( (x \text{ and } y) \), transport cost ratios \( (z) \), and inventory coverage \( (IC_1) \) at the Local DC, given a set of additional parameters – presented next – which were held constant. The basic idea was to investigate the relative importance of \( x, y, z, \text{ and } IC_1 \), providing initial insights on their behavior that could be cross-checked later with the simulation results, and also addressing validity issues regarding the conclusions obtained. In sequence, simulations were conducted on Eqs. (14)–(16), now allowing all parameters to vary within certain minimal and maximal ranges, in order to confirm the most relevant variables to the optimal distribution structure \( (W_{opt}) \) and to test the response of \( SS \) and \( TC \) to these variables, by means of further statistical testing. These steps are detailed next.

3.1. Parameters choice

The parameters for the sensitivity analyses were chosen by observing their mean values in the literature reviewed. They were collected from recent papers on inventory network planning (Wanke, 2009, 2010; Wanke and Saliby, 2009), which also provide a comprehensive compilation on their actual/most common ranges of variation, from minimal to maximal values,
found in previous international peer-reviewed papers. The parameters considered in the sensitive analyses are given in Table 1. The compilation of the actual parameters found in previous papers is reproduced in Appendix C.

### 3.2. Analytical model

With the purposes of investigating the impacts of different $x$, $y$, $z$, and $IC_1$ levels on the optimal distribution structure, as well as their relative importance, sensitivity analyses were conducted on $W_{opt}$ and $TC$ considering the mean values of the variables presented in Table 1 as starting points (additionally, $k_C$ was considered to be equal to 3.5). Replacing these values into Eqs. (15) and (16), it follows that $W_{opt}$ can be solely expressed in terms of $x$, $y$, $z$, and $IC_1$ and that $TC$ can be solely in terms of $x$, $y$, $z$, $IC_1$, and $W_{1,1}$. $TC$, for example, is now given by:

$$
TC = 1.19 \sqrt{15625y^2 + 3(100W_{1,1}(100W_{1,1}IC_1 - 100W_{1,1})) + 272.25(1 - W_{1,1})^2}x + 87.19W_{1,1}^2 + 25(1 - W_{1,1})z
$$

and, although the process to determine $W_{opt}$ is also straightforward, $W_{opt}$ is omitted here for the sake of simplicity, due to the large number of supporting variables – cf. Eqs. (17)–(24).

So, for a given set of parameters, the optimal distribution structure basically depends on: the inventory coverage level at the Local DC; the ratio between distribution and transference costs, from Central DC to market and from the Central DC to the Local DC; and the ratio between lead time means, from the factory to the Central DC and from the Central DC to the Local DC. The optimal distribution structure is, comparatively, less impacted by the ratio between lead time standard deviations, from the factory to the Central DC and from the Central DC to the Local DC.

Results presented in Fig. 4a–d indicate that the smaller the inventory coverage level at the Local DC the greater the percentage of echeloned distribution in the optimal network structure, particularly if the distribution costs from the Central DC to the market are substantially higher when compared to the transference costs from the Central DC to the Local DC. On the other hand, if the mean lead time from the factory to the Central DC is much longer than the mean lead time from the Central DC to the Local DC (suggesting a network structure where both DCs are relatively, closely located) the percentage of echeloned distribution is negligible.

In Fig. 4a–d, the contrasts for critical levels of $x$, $y$, $z$, and $IC_1$ reveal their relative importance on the optimal distribution structure. It can be seen that, when $IC_1$ is small, $y$ impacts much less than $x$ in favor of the echeloned distribution. However, even for higher values of $IC_1$, $z$ may play a special role in assuring a relevant share for the echeloned distribution within the optimal structure. Since $x$, $y$, $z$, and $IC_1$ not only interact differently with each other, but also with the remainder variables presented in Table 1, a numerical approach such as simulation is deemed necessary. This is discussed next.

### 3.3. Simulation

According to Evans and Olson (1998), Monte Carlo simulation (MCS) is basically a sampling experiment whose purpose is to estimate the distribution of an outcome variable that depends on several probabilistic input variables. In this research we are interested in the distribution of $W_{opt}$, $SS$, and $TC$ when demands, lead-times, costs, and other related inputs are random variables.

An important issue in MCS is choosing the number of replications of a simulation, since it affects the quality of the results. In general, the higher the number of replications, the more accurate will be the characterization of the output distribution and estimates of its parameters, such as the mean and the median. A common rule to determine the sample size ($n$) is to refer to the standard error of the sample mean ($SE$) and to the standard deviation ($\sigma$) of the individual values of the output variable (Evans and Olson, 1998). As a matter of fact, since $SE = \sigma/\sqrt{n}$, and although $\sigma$ is not known in advance, a sample size of 10,000 replications implies that $SE = 0.01\sigma$.

On the other hand, if we view the half-width ($\pm A$) of a confidence interval, set upon confidence level of $100(1 - \alpha)$%, as the accuracy we wish to obtain in estimating the mean, the following relationship should be observed: $A = z_{\alpha/2}\sigma/\sqrt{n}$. Supposing a confidence of at least 0.99 assurance ($z_{0.995} = 2.575$) and rewriting $\sigma$ as $\mu CV$, that is, the product of the mean of the

<table>
<thead>
<tr>
<th>Parameters used.</th>
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<tbody>
<tr>
<td>Variables</td>
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<tr>
<td>$D_1$ (units per day)</td>
</tr>
<tr>
<td>$s_{1,1}$ (units per day)</td>
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<tr>
<td>$LT_1$ (in days)</td>
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<td>$s_{1,1}$ (in days)</td>
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<td>$h$ ($$/unit/day)</td>
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individual values of the output variable ($\mu$) by its coefficient of variation ($CV$), one easily gets that $A = 0.02575\mu CV$. Even for a extremely high $CV$, say 5, the half-width $A$ is still almost eight times smaller than $\mu$.

So, balancing the estimation benefits of a large sample size with its processing times, a MCS with 10,000 replications was conducted: (a) to confirm the most relevant variables to the optimal distribution structure, identifying under what circumstances a direct, an echeloned, or a mixed distribution structure should be preferred; (b) to test the response of SS and TC to these variables.

Local DC and Central DC present equivalent lead time standard deviations ($y = 1$) and distribution costs ($z = 1$)

Direct distribution is preferable, regardless the inventory coverage level at the Local DC ($W_{opt} < 0$)

Local DC and Central DC present equivalent lead time means ($x = 1$), standard deviations ($y = 1$), and distribution costs ($z = 1$)

When the ratio between lead time means from factory to Central DC and from Central DC to Local DC equals 15, the optimal structure consists of a marginal percentage of echeloned distribution ($W_{opt} close to zero$)

However, this percentage is higher when the inventory coverage level at the Local DC ($IC_1$) is lower

Fig. 4. (a–d) Contrasts for $x$, $y$, $z$, and $IC_1$ given the mean values presented in Table 1.
Again, the particular case where $n = m = 1$ was considered, similarly to the previous section. The simulation was performed with the aid of an Excel spreadsheet and each one of the rows of the spreadsheet corresponded to a single replication. For each one of the 10,000 replications, uniformly distributed random parameters were generated based on minimal and maximal values presented in Table 1. In addition to these parameters, $x$, $y$, $z$, and $IC_1$ ranged uniformly between 1 and 30 and $k_c$ ranged uniformly between 2.5 and 4.5.

Local DC and Central DC present equivalent lead time means ($x = 1$) and standard deviations ($y = 1$)

When the ratio between distribution costs from Central DC to market and from Central DC to Local DC equals 6 ($z$), the optimal structure consists of a substantial percentage of echeloned distribution ($W_{opt}$ gets higher than 0.15), if the inventory coverage at the Local DC is low ($IC_1$)

The greater the inventory coverage at the Local DC, the smaller this percentage

Local DC and Central DC present equivalent lead time means ($x = 1$) and distribution costs ($z = 1$)

Even when the ratio between lead time standard deviations from factory to Central DC and from Central DC to Local DC equals 30 ($y$), the optimal structure still consists of direct distribution ($W_{opt} < 0$)

Fig. 4 (continued)
The values obtained for these variables constituted the columns of the spreadsheet, with 10,000 observations each. These values were then substituted into the $W_{opt}$, $SS$, and $TC$ equations, in order to allow the collection of the respective outcomes for each replication. It should be noted, that like previous studies, continuous uniform distributions, independent from each other, were considered in the random number generation.

3.3.1. Optimal distribution structure
The distribution of frequencies for $W_{opt}$ is displayed in Fig. 5. The optimal distribution structure is asymmetrically distributed around its mean. In 7419 of the 10,000 scenarios, the optimal solution for the distribution structure ranged within values greater than 0 and smaller than 1 (mixed distribution structure). For the remainder scenarios, the optimal solution was 0 and 1; totaling 1732 (direct distribution) and 849 (echeloned distribution) cases respectively.

Non-parametric tests were conducted to explore the differences in medians among the three groups of responses: direct, echeloned, and mixed distribution structures. Not only the variables presented in Table 1 were tested, but also $x$, $y$, $z$, $IC_1$, and $k_C$. The results for the Kruskall–Wallis’ Tests are shown in Table 2 and significant differences are flagged ($p < 0.05$).

In general, echeloned and mixed distribution structures tend to present lower total costs when compared to the direct ones (frequently characterized by higher inventory holding cost items and distribution costs). However, network safety stock levels are substantially smaller under direct and mixed distribution structures than under echeloned distribution structures (inventory decentralization with no possibility of pooling lead time demands). The rationale behind these issues is addressed next.

The ratio between distribution costs, from Central DC to market, and transference costs, from Central DC to Local DC, is smaller under direct distribution structures than under echeloned and mixed distribution structures. As this ratio goes up,
In the optimal structure moves from direct to mixed distribution and then to echeloned distribution, where the transference costs from Central DC to Local DC are extremely competitive when compared to direct distribution costs.

One can appreciate the remarkable impact of the inventory holding costs $C_i$ on the optimal distribution structure. As expected, high holding costs items should be kept centralized (direct distribution), and, the smaller the holding cost, the higher is the degree of inventory decentralization through the Local DC. More precisely, as per-unit inventory holding costs go down, one should move from direct to mixed distribution structures and then to echeloned distribution, where all demands are served by Local DCs.

Inventory coverage at the Local DC is substantially smaller under echeloned distribution when compared to mixed distribution. This occurs because inventory coverage positively impacts the replenishment variance allocated to the Central DC. In other words, the higher the inventory coverage at the Local DC, the higher the replenishment variance to be met by safety stocks at the Central DC. However, differently from echeloned or even direct distribution, under mixed distribution structures it is possible to pool lead time demands from different sources at the Central DC. As a direct consequence of the compensation of fluctuations of demand during lead time, mixed distribution structures turns out to be more adequate where lead time means and demand/replenishment variances are higher when compared to direct or echeloned structures. Conversely, when the inventory coverage at the Local DC is low, the adoption of an echeloned distribution structure may be pertinent to avoid the impacts of undesirable lead time demand/replenishment variability at the Central DC.

Therefore, one must decide between adopting (1) a direct distribution structure for high inventory holding cost items with competitive distribution costs; (2) an echeloned distribution structure for low inventory holding cost items and prohibitive direct distribution costs (by means of a Local DC located relatively far from the Central DC and small inventory coverage to isolate from the market the high lead time variability to serve the Central DC); or (3) a mixed distribution structure for intermediate holding cost items and direct distribution costs (by means of a Local DC located relatively far from the Central DC and large inventory coverage to balance the high demand/replenishment uncertainty from serving the markets and the Local DC with the low lead time variability from the factory to the Central DC).

### 3.3.2. Impact on total costs and network safety stock levels

Linear models for $SS$ and $TC$ were developed taking into account all the independent variables presented in Table 2. More precisely, an ordinary least-squares (OLS) regression method was performed with SPSS for both dependent variables; results are presented in Table 3 (only significances below 0.05 were flagged).

As one can see, both models present a good explanatory power for $SS$ and $TC$ ($R^2 > 0.80$), and the ratio between lead time standard deviations ($y$), from the factory to the Central DC and from the Central DC to the Local DC, is the most significant variable in both models.

This variable plays a pivotal role in determining network safety stock levels, since its standardized coefficient is greater than the summation of the coefficients of the other variables. The magnitude of the lead time standard deviation ratio not only indicates opportunities for lead time demand pooling (via mixed distribution structures) when it is moderate, but also points to the need to isolate the Central DC (via echeloned distribution structures) when it is high enough. As a matter of fact, $y$ is the underlying variable regarding the mission of the Local DC within an echeloned or a mixed distribution structure, determining whether it should be an instrument for balancing lead times and demands uncertainties or for isolating the Central DC lead time variability from the market and replenishment uncertainties.

On the other hand, when total costs are analyzed, the inventory holding cost, the lead time standard deviation to serve the Local DC, and the service level factor at the Central DC should also be taken into consideration altogether with $y$. These four

| Table 3 | Linear models for $SS$ and $TC$. |
|-----------------|-----------------|-----------------|-----------------|
| | Network safety stock levels ($SS$) | Total costs ($TC$) |
| | Unstandardized coefficients | Std. coef. | $t$ | Sig. | Unstandardized coefficients | Std. coef. | $t$ | Sig. |
| | $B$ | Std. error | Beta | $t$ | Sig. | $B$ | Std. error | Beta | $t$ | Sig. |
| (Constant) | $-17402.251$ | $197.577$ | $-88.078$ | **.000** | (Constant) | $-9249.787$ | $127.959$ | $-72.287$ | **.000** |
| $y$ | $398.025$ | $1.271$ | $.773$ | $313.073$ | **.000** | $y$ | $140.897$ | $.823$ | $.573$ | $171.119$ | **.000** |
| $s_{LT,1}$ | $4835.972$ | $25.067$ | $.477$ | $192.925$ | **.000** | $h$ | $6178.967$ | $38.335$ | $.543$ | $161.184$ | **.000** |
| $k_c$ | $1682.420$ | $18.767$ | $.221$ | $89.650$ | **.000** | $s_{LT,1}$ | $1737.496$ | $16.234$ | $.358$ | $107.027$ | **.000** |
| $D_t$ | $64.015$ | $1.862$ | $.085$ | $34.375$ | **.000** | $k_c$ | $634.733$ | $12.154$ | $.175$ | $52.194$ | **.000** |
| $h$ | $-1174.067$ | $59.191$ | $-.049$ | $-19.835$ | **.000** | $z$ | $1641.941$ | $.828$ | $.060$ | $20.481$ | **.000** |
| $z$ | $1641.941$ | $1.279$ | $.033$ | $13.249$ | **.000** | $D_t$ | $24.671$ | $1.206$ | $.068$ | $20.456$ | **.000** |
| $C_{LT,1}$ | $956.186$ | $125.432$ | $.019$ | $7.623$ | **.000** | $C_{LT,1}$ | $1074.896$ | $81.235$ | $.044$ | $13.232$ | **.000** |
| $l_C_1$ | $-3.364$ | $1.287$ | $-.006$ | $-2.613$ | **.009** | $l_C_1$ | $4.066$ | $.834$ | $.016$ | $4.877$ | **.000** |
| $x$ | $-2.934$ | $1.279$ | $-.006$ | $-2.293$ | **.022** | $x$ | $3.486$ | $.829$ | $.014$ | $4.207$ | **.000** |
| $k_N$ | $-31.676$ | $18.711$ | $-.004$ | $-1.693$ | **.090** | $s_{LT,1}$ | $1.971$ | $.893$ | $.007$ | $2.206$ | **.027** |
| $s_{LT,1}$ | $1.881$ | $1.379$ | $.003$ | $1.364$ | **.173** | $LT_t$ | $10.128$ | $.604$ | $.006$ | $1.676$ | **.094** |
| $LT_t$ | $-11.070$ | $9.328$ | $-.003$ | $-1.187$ | **.235** | $k_N$ | $10.931$ | $.1218$ | $.003$ | $.902$ | **.367** |
| Adj. R square | $.889$ | $F$ | $12155.400$ | Sig. | **.000** | Adj. R square | $.803$ | $F$ | $5929.427$ | Sig. | **.000** |
variables relate to the inventory impacts caused by a Local DC: not only would additional safety stock levels be necessary at this facility, they would also be necessary at the Central DC due to higher demand/replenishment variances. It should be noted, yet, that the next three variables on the list, \( z \), \( D_1 \), and \( C_{e1} \), are transportation cost-related variables. However, their smaller impact on total costs can be attributed to the parameters used in this analysis and to the experiment performed within a simpler network \((n = m = 1)\).

4. Managerial implications

From a managerial perspective, Table 4 indicates the adequacy of direct, echeloned, and mixed distribution structures to different characteristics of product, operation, and demand. Table 4 also provides guidance on major operational thresholds that can arise during the decision of the most adequate distribution structure, such as the convenience of lead time demand pooling and, therefore, the mission of a Local DC. Impacts on total costs and network safety stock levels are also addressed.

With respect to mixed distribution structures, of note is the increasing number of supply chains offering to end customers the possibility to buy online and receive the product directly from a central warehouse at home, in parallel to the traditional purchase at physical stores. Not only have certain bookstore chains allowed both purchasing alternatives to customers, but Dell also offers the alternatives of buying online (customized computers) or at Walmart (standardized models).

Therefore, the growing number of B2C and B2B initiatives may limit purely direct and echeloned distribution structures to very particular cases, where the inventory holding costs and the ratio between direct distribution and transference costs completely offset the benefits derived from lead time demand pooling. The oil industry consists of a typical case where purely echeloned distribution prevails through refineries, fuel bases, and service stations (low holding cost items altogether with prohibitive direct distribution costs). On the other hand, expensive slow-moving spare parts may benefit from direct distribution by air to end customers, since the inventory holding costs and shortage risks are more relevant than outbound transportation costs.

5. Conclusions

This paper explored the impact of three different distribution structures on total costs and network safety stock levels, as well as their suitability to different characteristics of the product, operation, and demand.

The contributions of the paper to the literature are fourfold. First, this research differs from previous studies by explicitly considering a third possible distribution structure – here called mixed – besides the purely direct and echeloned ones. Second, differently from previous researches where stochastic, base-stock level models were used as analytical cornerstones, the analyses developed here are built upon inventory portfolios and consolidation effects formulae. Third, inventory portfolios and consolidation effects formulae were expanded, by using Chebychev’s inequality for safety stock dimensioning, to handle with the case where Local DCs are served by one Central DC. Fourth, this paper also differs from previous work by presenting a managerial framework to help in deciding the most adequate distribution structure. Sensitivity analyses performed via mathematical expressions and numerical simulation, used to develop this framework, indicate that:

- inventory holding costs remarkably impact the distribution structure: high holding cost items should be directly distributed to markets and, as these costs keep going down, mixed and echeloned distribution structures should be considered in sequence;

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Variables</th>
<th>Distribution structures</th>
</tr>
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<tbody>
<tr>
<td>Adequacy</td>
<td>Inventory holding costs</td>
<td>Direct</td>
</tr>
<tr>
<td></td>
<td>Ratio between distribution and transference costs (^a)</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>Ratio between lead time means(^b)</td>
<td>Medium/ high</td>
</tr>
<tr>
<td></td>
<td>Ratio between lead time standard deviations(^c)</td>
<td>Medium</td>
</tr>
<tr>
<td>Operational thresholds</td>
<td>Inventory coverage at Local DC</td>
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</tr>
<tr>
<td></td>
<td>Lead time demand pooling?</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>Local DC mission</td>
<td>N/A</td>
</tr>
<tr>
<td>Impact</td>
<td>Network safety stocks</td>
<td>Medium</td>
</tr>
<tr>
<td></td>
<td>Total costs</td>
<td>High</td>
</tr>
</tbody>
</table>

\(N/A\) – non-applicable.

\(^a\) Low ratio indicates that direct distribution costs are competitive.

\(^b\) High ratio indicates a Local DC relatively, closely located to the Central DC.

\(^c\) Low ratio indicates potential for lead time demand pooling.
• the ratio between direct distribution and transference costs is a countervailing force that should be analyzed simultaneously with inventory holding costs;
• the ratio between lead time standard deviations, from factory to Central DC and from Central DC to Local DC, is the major variable to determine the mission of the Local DC under a mixed and an echeloned distribution structure, that is, respectively, to pool demands and lead times uncertainties or to isolate them;
• the mission of the Local DC is strongly associated with its inventory coverage and its location: one should decide between a Local DC closely located to the market and small inventory coverage (echeloned structure) and a Local DC closely located to Central DC and large inventory coverage (mixed structure).

As regards future research endeavors, one possible extension of this research could be the incorporation of some of the distribution structure analyses presented here into larger multi-echeloned logistics network problems. The basic idea is to determine, via nonlinear programming, how these structures are reflected in the optimal network design and how such design might relate to a given set of product, demand, and operation characteristics. In such network problems, the trade-offs between mixed distribution structures and the practice of regular transshipments are issues that are particularly relevant and the subjects of ongoing future research.

Appendix A

Chebychev’s inequality provides a relationship between the standard deviation and the dispersion of the probability distribution of any random variable (Montgomery and Runger, 1994). Originally applied in inventory management by Eppen and Martin (1988), its result is interpreted as follows. The probability (Pr) that a random variable \(D\) differs from its mean \(\mu\) by at least \(k\) standard deviations \(\sigma\) is less than or equal to \(\frac{1}{k^2}\). That is:

\[
\Pr(D > k\sigma) \leq \frac{1}{k^2}.
\]  

(A1)

Appendix B

The variance of the replenishment orders \(V_{RC,I}\) allocated to the Central DC, generated by the requests of a given Local DC \(l\), can be determined based on \(Q_l\). Let \(p_l\) be the probability of the Local DC \(l\) requesting a lot size \(Q_l\) from the Central DC on a given day and \((1 - p_l)\) the probability of not requesting this lot (i.e., a null request). Assuming that a good approximation for \(p_l\) is the inverse of the inventory coverage level at the Local DC \(l\) \((IC_l\text{, measured in days of consumption})\), it follows that:

\[
p_l = \frac{1}{IC_l} = \frac{\sum_{i=1}^{n} W_{li} D_i}{Q_l}.
\]  

(A2)

and

\[
(1 - p_l) = \frac{Q_l - \sum_{i=1}^{n} W_{li} D_i}{Q_l}.
\]  

(A3)

Thus, \(V_{RC,I}\) is simply given by:

\[
V_{RC,I} = E[R_{C,l}^2] - E[R_{C,l}]^2,
\]  

(A4)

where \(R_{C,I}\) is the random variable denoting the replenishment to the Local DC \(l\) allocated to the Central DC. Then:

\[
V_{RC,I} = \left[Q_l^2 p_l + 0^2(1 - p_l)\right] - \left[Q_l p_l + 0(1 - p_l)\right]^2 = \left[Q_l^2 p_l\right] - \left[Q_l p_l\right]^2,
\]  

(A5)

\[
V_{RC,I} = \left[Q_l^2 p_l\right] - \left[Q_l p_l\right]^2, \quad \text{(A6)}
\]

\[
V_{RC,I} = \left(\sum_{i=1}^{n} W_{li} D_i\right) \left(Q_l - \left(\sum_{i=1}^{n} W_{li} D_i\right)\right).
\]  

(A7)

Appendix C

See Table C1.
Table C1
Actual parameters compiled from previous papers (adapted from Wanke (2009, 2010).

<table>
<thead>
<tr>
<th>Papers</th>
<th>Parameters</th>
<th>D</th>
<th>LT</th>
<th>$s_D$</th>
<th>$s_{LT}$</th>
<th>k</th>
<th>i (%)</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tallon (1993)$^a$</td>
<td>500; 650; 900; 1000; 1200</td>
<td>1; 2; 3; 5</td>
<td>20; 25; 100; 200; 250</td>
<td>0.25; 0.50</td>
<td>i; 2</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Caron and Marchet (1996)$^b$</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Evers (1997)$^c$</td>
<td>100; 300; 500</td>
<td>5; 10; 15</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tyagi and Das (1998)$^d$</td>
<td>50; 100</td>
<td>5; 10</td>
<td>1; 3; 10</td>
<td>0; 0.5</td>
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<tr>
<td>Evers (1999)$^e$</td>
<td>77; 350; 750</td>
<td>6</td>
<td>25; 150; 300</td>
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<td>25</td>
<td>0.75; 2.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ballou and Burnetas (2003)$^f$</td>
<td>100</td>
<td>5</td>
<td>10</td>
<td></td>
<td>25; 0.75; 2.5</td>
<td></td>
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</tbody>
</table>

References


